

Math: Perceptron Squared

Problem Story

Alice has been assigned a new task by the Lemon AI team: implement a model that predicts RAM module prices based on the current time.

Ivan had already implemented such a model, but unfortunately, the source code was lost and he can't recall the implementation details. Luckily, a compiled version of the code still exists.

After some experimentation, Alice determined that the model's output is $g(t) = 4.5t + 8$, where t is the number of days since the start of 2026.

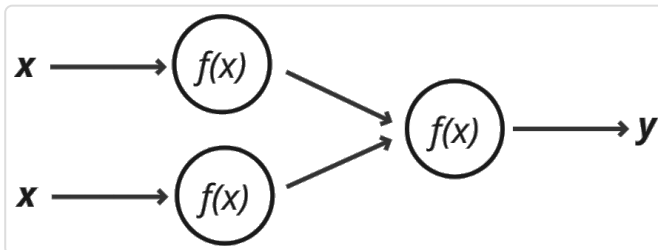
She also discovered that the model is a two-layer Multi-Layer Perceptron (MLP).

Multi-Layer Perceptron (MLP) — a neural network consisting of multiple layers of neurons. Each neuron receives inputs, applies weights to them, sums the weighted values, and passes the result through an activation function (a mathematical transformation). The output of one layer becomes the input for the next layer.

Problem

The MLP used in Ivan's model has a simple architecture:

- All weights are equal to 1
- The first layer has two neurons, and the second layer has one neuron
- Each neuron uses the same linear activation function $f(x) = ax + b$



Since the team plans to extend and improve the model in the future, Alice needs to reimplement the original MLP architecture rather than simply using the function $g(t)$ directly. To do this, she must determine the activation function $f(x) = ax + b$. Help Alice find the values of a and b !

The function f is of the form $f(x) = ax + b$. It is known that $f(2f(x)) = 4.5x + 8$. Find the original function f in terms of the values a and b .

Task {1} Find a

Task {2} Find b

Grading

- 50% for correct {1}
- 50% for correct {2}

Further explanation

You can solve the problem with no knowledge on MLPs. But if you are interested and want to find out more about them, here is short video by IBM on Multi-Layer Perceptrons: <https://www.youtube.com/watch?v=7YaqzpitBXw>